

METHODS OF SPATIAL POINT PATTERN ANALYSIS APPLIED IN FOREST ECOLOGY

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Spatial patterns of forest trees result from complex dynamic processes such as establishment, dispersal, mortality, land use and climate (Franklin et al. 2010), especially in tropical forests which are among the world's most species-rich terrestrial ecosystems. Spatial correlation of trees may provide evidences of ecological interactions which are assumed to be drivers of spatial pattern in plant communities. Spatial pattern analysis in ecology has received increasing attention of ecologists and mathematicians over the last decades. Furthermore, it is stimulated by the development of spatial point pattern methods and relevant computer applications.

Several processes and mechanisms have been proposed to explain species coexistence and community structure. For example, plant-plant interactions, such as competition or facilitation (Bruno et al. 2003), limited dispersal (Nathan & Muller-Landau 2000), habitat preference (Harms et al. 2001), density dependent mortality (Janzen 1970; Connell 1971) and neutral theory ((Hubbell 2001; Chave 2004). Hence, understanding these underlying processes is a central goal in ecology (Tilman 1994).

The neutral theory is proposed in order to find an explanation for the observed patterns of species abundance across scales in space and time (Chave 2004). This theory assumes that all individuals in a community are strictly equivalent regarding their prospects of reproduction and death (Chave 2004). However, there is ample evidence showing that species are not equivalent and differ in their ecological traits (Wiegand et al. 2007).

Janzen(1970) and Connell(1971) hypothesized that host specific pests reduce recruitment near conspecific adults, thus freeing space for other plant species. Condit et al. (1994) suggested that Janzen-Connell hypothesis is exhibited basically among those species with the highest population densities, other studies show that density dependence is very common in tropical tree species (Lan et al. 2009). Moreover, it is expected that aggregation should decrease with increasing tree size (age) classes, due to competition (Sternler et al. 1986).

Callaway & Walker(1997) stated that competition has long been recognized as a main force in structuring plant communities, while facilitation has not received much attention. They found that the relative importance of these two processes can be evaluated by investigating the effects of abiotic stress, consumer pressure, life stage, age and intensity of interaction strengths. Facilitation has been observed to increase establishment of seedlings close to adults such as nurse plants (Lan et al. 2012) or mycorrhizal fungi (Dickie et al. 2007). The species herd protection hypothesis is an extension of the Janzen-Connell hypothesis (Peters 2003) and suggests that hetero-specific neighbors can promote species coexistence by thwarting the transmission of biotic plant pests (Lan et al. 2012). Thus, major mechanisms or processes leading to aggregation or over-dispersion of plant distribution still remain controversial (Murrell 2009). However, tree species are long-lived, therefore long-term observations including period censuses are needed to examine the effects of competition and facilitation in forest community ecology.

Colonization limitation, which is also called recruitment or dispersal limitation, is an important factor in successional dynamics, community diversity and composition (Tilman 1994). Seeds can be dispersed in various spatial scales depending on the specific mechanisms or agents, e.g. by wind, animals and/or gravity. Seed dispersal patterns vary among plant individuals, species and populations and differ in distances from parents, micro-sites and times (Nathan & Muller-Landau 2000).

Niche differentiation is a prominent hypothesis explaining the maintenance of tree species diversity in tropical forests (Connell 1978). It suggests that different species are best suited to different habitats in which they are completely dominant and more abundant than in less suitable habitats (Harms et al. 2001). In addition, distribution patterns of tropical trees are generally more clumped or aggregated than random (Condit et al. 2000). Furthermore, environmental heterogeneity (difference in soil, elevation, slope, etc.) may obscure aggregated distribution of plant species across spatial scales (Harms et al. 2001). Therefore, it is difficult to assess whether aggregated patterns are caused by local dispersal, local interaction or environmental heterogeneity.

I. METHODS

Point processes are stochastic models of point patterns while a point pattern is a collection of points which is typically interpreted as a sample from a point process (Diggle 2003). The fundamental difference between the two terms is that a point process is a theoretical stochastic model or random variable, whereas a pattern is a realization of the process (Perry et al. 2006). In a simple case, each point pattern is defined by sets of Cartesian coordinates (x_i, y_i) and referred to events. Moreover, an additional property of an event, a so-called mark, can be attached. Therefore, a set of events typically takes the form $\{(x_i, y_i, m_i)\}$, giving the locations (x_i, y_i) and marks m_i in the region of observation (Stoyan & Penttinen 2000). For example, mapped data of trees contain the positions of stems and the marks (such as species, diameter, height, etc.).

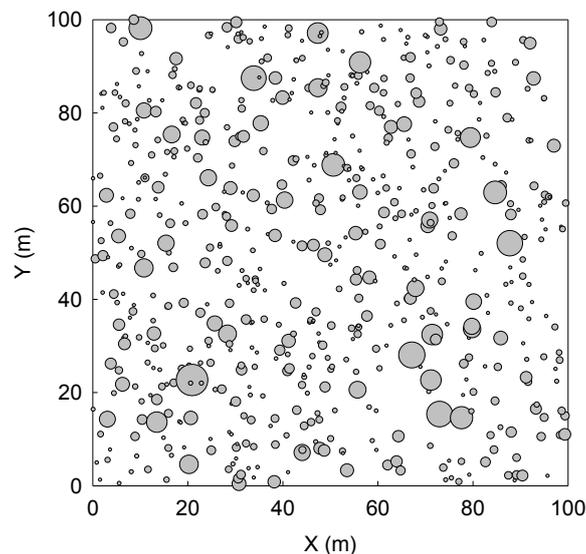


Fig. 1: A map of trees with locations and diameters proportional to size of circles

A fundamental property of a point process is the point intensity λ , which may be interpreted as the mean number of points per unit area. A point process N is called homogeneous

(stationary) if N and its translated point processes have the same distribution for all translations (Diggle 2003). The simplest case is complete spatial randomness (CSR) and termed the homogeneous Poisson process with intensity λ . This point process has two important properties (Stoyan & Penttinen 2000): (1) the number of events in any area unit follows the Poisson distribution with mean λ . (2) Given n events in the observation region, their positions follow an independent sample from the uniform distribution in this region.

A spatial point pattern can be characterized by its first-order and second-order properties. The first-order property describes the mean number of events per unit area while the second-order property is related to the variance of the number of events per unit of observed area (Perry et al. 2006). A point pattern may deviate from stationarity in cases (Diggle 2003): (i) The intensity function $\lambda(x, y)$ or point density is not constant but varies spatially; (ii) The local point configurations may be location-dependent. This generalizes to the inhomogeneous Poisson process in which the constant intensity in CSR is replaced by an intensity function $\lambda(x, y)$ whose value varies with the location (x, y) (Diggle 2003). The different intensities are shown in fig. 2.

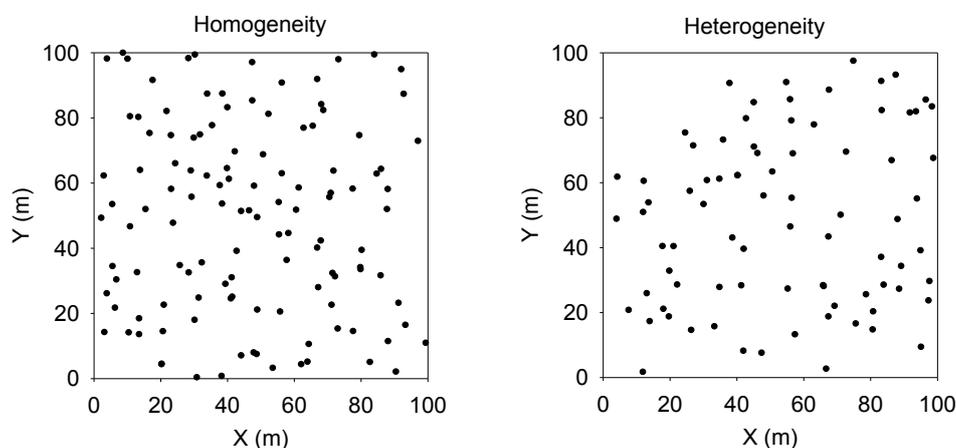


Fig. 2: Examples of spatial distributions of points with constant and varying intensities. **Ripley's K-function and pair-correlation function**

The Ripley's K -function is the expected number of points in a circle of radius r around an arbitrary point, divided by the intensity λ of the pattern (Ripley 1976). Thus, Ripley's K is cumulative up to distance r meaning that point intensity is calculated within entire circle with radius r .

$$K(r) = 2\pi \int_{t=0}^r g(t)tdt$$

$\lambda K(r)$ is the mean number of points within a distance r from an arbitrary point, particularly $K(r) = \pi r^2$ for a homogeneous Poisson process. Let $L(r) = (K(r)/\pi)^{0.5} - r, r \geq 0$; thus $L(r) = 0$ for a homogeneous Poisson process; i.e., a straight line with slope 0 (Mateu 2000).

For computation of the pair-correlation function $g(r)$, the circle is replaced by a ring. $g(r)$ is the expected density of points at distance r from an arbitrary point, divided by the intensity λ of the pattern (Stoyan & Stoyan 1994). The difference in computation of K - and g -functions is presented in fig. 3. Therefore, we can determine whether a pattern is random, clumped or regular at a specific distance r . Under CSR, $g(r) = 1$, under aggregation $g(r) > 1$ and under regularity $g(r) < 1$. An example is shown in fig. 4.

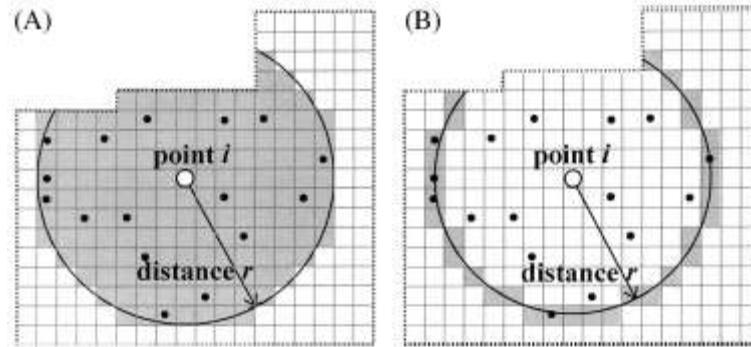


Fig. 3: The difference in computation between K -function (A) and g -function (B)

Both Ripley's K -function and the g -function can be used as bivariate functions when considering the spatial relation of 2 point patterns. Hence, the bivariate K -function $K_{12}(r)$ is defined as the expected number of points of pattern 2 within a given distance r of an arbitrary point of pattern 1, divided by the intensity λ_2 of points of pattern 2. Similarly, the pair-correlation function $g_{12}(r)$ gives the expected density of points of pattern 2 at distance r from an arbitrary point of pattern 1, divided by the intensity λ_2 of points of pattern 2 (Wiegand & Moloney 2004). $g_{12}(r)$ indicates whether pattern 2 is characterized by (1) independence ($g_{12}(r) = 1$), (2) repulsion ($g_{12}(r) < 1$) and (3) attraction ($g_{12}(r) > 1$) from pattern 1 at distance r . The pair-correlation function $g_{12}(r)$ is related to $K_{12}(r)$ by:

$$K_{12}(r) = 2\pi \int_{t=0}^r g_{12}(t)t dt$$

Null models and hypothesis testing

To answer specific biological questions related to dynamics of plant distribution, one may test the observed data based on an appropriate null hypothesis to find departure from the null model. To choose an appropriate null model, a proposed approach is based on the mathematical form of $K(r)$ or $g(r)$ functions: (i) inspection of the estimated $K(r)$ or $g(r)$ to find appropriate models and parameters for the point process, (ii) construction of confidence envelopes via Monte Carlo simulations of the stochastic process (Wiegand & Moloney 2004).

There are two commonly used null models for simulating a univariate point pattern: complete spatial randomness (CSR) and heterogeneous Poisson process (HP). The CSR null model is implemented as a homogeneous Poisson process where the intensity λ is constant over the study region (Wiegand & Moloney 2004). Inversely, the HP null model is applied when a point pattern is not homogeneous, therefore varying values of point intensity are quantified by a function $\lambda(x,y)$.

For analyzing a bivariate point pattern, two null models are mainly used: independence and random labeling. The independence null model assumes two patterns are generated by two different processes, and therefore is used to test the independence of two point patterns, for example two point patterns of two different tree species. The random labeling null model hypothesizes that two patterns are created by the same stochastic processes, for instance two point patterns of a tree species (e.g. dead and alive). In practice, Goreaud & Pelissier(2003) gave detailed suggestions when to use which null model and how to avoid misinterpretations. Alternatively, antecedent conditions may be useful to choose as an appropriate null model in

some practical cases. In this null model, pattern 1 (e.g., adult trees) is kept unchanged but for the locations of pattern 2 (e.g., saplings) is randomized (CSR is assumed), because adults do not change their positions over time but saplings may be found in the entire observed region.

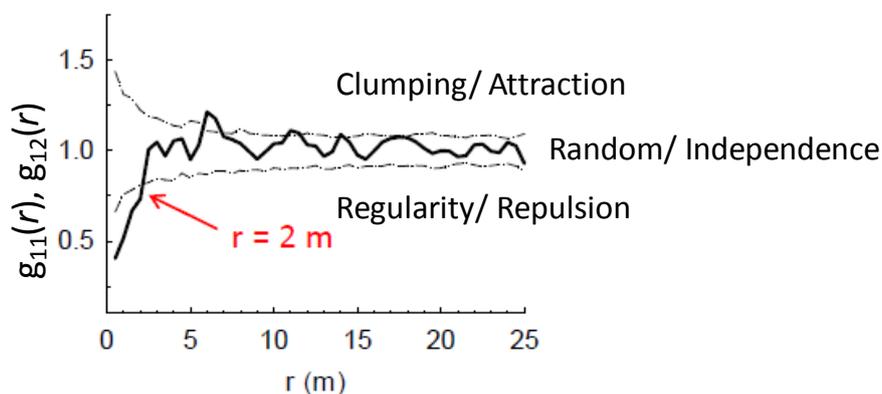


Fig. 4: Typical forms of pair-correlation functions $g_{11}(r)$ and $g_{12}(r)$

A principal advantage of Monte Carlo testing is that the investigator is not constrained to know distribution theory and can use informative statistics (Diggle 2003). Once distribution theory is known, Monte Carlo testing can be used to check its applicability. Due to mathematically unknown or intractable distribution theory of stochastic point processes, significance tests for spatial measures are often carried out by Monte Carlo simulation (Diggle 2003; Perry et al. 2006). Based on a null hypothesis, the data sets were simulated by calculating the statistic values (Marriott 1979) and rejection limits via confidence envelopes were estimated. Wiegand & Moloney(2004) provided a detailed guideline for choosing an appropriate null model for observed point data.

Models for marked point processes

Marks are used as properties of the objects (e.g., trees) and may be qualitative (e.g., species, damage level) or quantitative (e.g., diameter of tree, tree height). Therefore, marked point processes are models for random point patterns where marks are attached to the points (Illian et al. 2008). In mathematical notation, a marked point process M is a sequence of random marked points, $M = \{[x_n; m(x_n)]\}$, where $m(x_n)$ is the mark of the point x_n .

Similar to pure point processes, marked point processes can be used to consider relationships of two types of marks (Mateu 2000). We consider qualitatively marked point processes as sub-processes of point processes for aggregation or repulsion to find correlation structure in the marks, conditional on spatial pattern of the trees carrying the marks. The analysis of quantitative marks addresses questions concerning the numerical difference among the marks that is dependent on the distances of the corresponding points, for example, why neighboring points tend to have smaller (larger) marks than the mean mark (Getzin et al. 2008; Getzin et al. 2011).

The mark correlation function $k_{mm}(r)$ is defined as $k_{mm}(r) = c_{mm}(r)/\mu^2$ for $r > 0$, where $c_{mm}(r)$ is the conditional mean of the product of the marks of a pair of points with distance r ; μ is the mean mark (Illian et al. 2008). This normalization allows comparing the strength of mark correlation between different processes. If the empirical mark correlation functions are not constantly equal 1, there is reason to assume that marks are not independent. Applied to forest

ecology, $k_{mm}(r) < 1$ is assumed to indicate inhibition, individual trees compete against each other and thus have smaller than average marks if they are distance r apart. Inversely, $k_{mm}(r) > 1$ indicates that points at distance r have average marks larger than the mean mark.

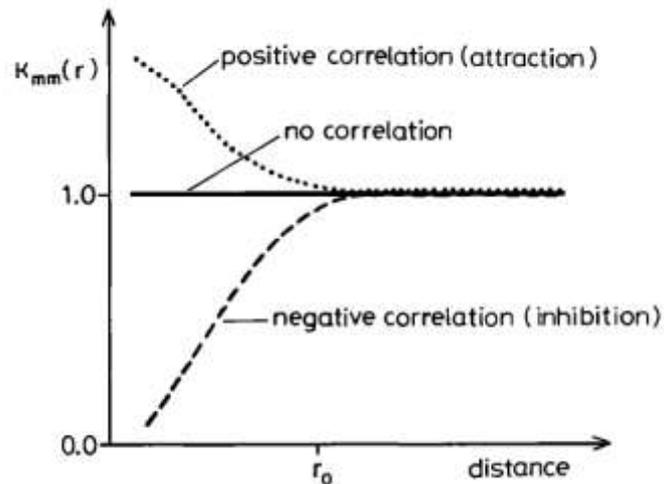


Fig. 5: Typical forms of the mark correlation function

II. DISCUSSION

Even though applications of point process methods have been developed and widely implemented in various scientific fields, these tools are bounded in practical uses. There are three main reasons: (1) requirement of mapped data, (2) pairwise-based second-order characteristics, and (3) snapshot analysis of pure spatial patterns (Comas & Mateu 2007). Basically, in point process models and spatial statistical tools, pair-wise analysis is a major part of second-order characteristics and analysis tools for multi-specific interaction do not analyze more than two variables. However, spatial correlation provides a sensitive indicator of ecological interactions structuring spatial patterns of plant species in communities (Wiegand et al. 2007). Moreover, snapshot observations combined with time series analyses have specific advantages (e.g., less time consuming and cheaper) and are appropriate approaches for dynamic assessments of long-lived tree species (Wiegand et al. 2000; Halpern et al. 2010).

In spatial point patterns analysis, an observed spatial pattern from the K -statistics is compared to a hypothetical model and evaluated via confidence envelopes, which are constructed by the maximum and minimum results computed across the simulated patterns. However, results from this approach are problematic because of violation of Monte Carlo methods and incorrect type I error performance rate (Loosmore & Ford 2006). A proposed solution is goodness-of-fit test as implemented in the software Programita (<http://programita.org/>). However, other authors stated that Monte Carlo method is appropriate and can be used to assess whether the spatial pattern is significantly different from random (Dale et al. 2002).

In computing point-pattern statistics, edge correction is required since events near the edge of the study region have fewer neighbors than centered events, leading to incorrectly calculated-intensities. Therefore, circle or ring samples will produce a biased estimation of the point pattern if used without edge correction (Wiegand & Moloney 2004). Three approaches are proposed for dealing with edge effect: Ripley's weighted correction, a toroidal correction and a guard area

correction (Haase 1995; Yamada & Rogerson 2003). The major finding of Yamada & Rogerson(2003) is that the K -function method adjusted by either the Ripley or toroidal edge is more powerful than the guard area method. Among two alternatives to estimate the bivariate K -function (numeric and analytical methods), numeric approaches are integrated and use an underlying grid of cells. Therefore, analyses using Programita software do not require edge correction (Wiegand & Moloney 2004).

Generally, plant community dynamics are driven by spatially dependent birth, death and growth processes and closely embedded in a heterogeneous landscape (Law et al. 2009). From multispecies spatial patterns analyzed, ecologists may use spatio-temporal information to tackle basic questions. Moreover, plants are obviously not points, marks characterized for individual plants (e.g., species, biomass, height, so on) and environmental data need to be considered as they are highly relevant (Illian & Burslem 2007). Therefore, theoretical and empirical issues are closely connected with their estimations and infer to the real dynamic processes generating patterns of plant communities. Spatial point pattern analysis is stimulated by large and technical literature in mathematics and plant ecology, moreover by strongly developed applications in computer science (Law et al. 2009).

III. CONCLUSION

Analysis of spatial point pattern has a long history in plant ecology and there are a number of tests available to characterise and explore such data. However, these tests do not all perform equally and all have their weaknesses and strengths. As a result, it is suggested that a suite of statistics is used to characterise spatial point patterns, otherwise there is a risk that the description of the pattern will be partially determined by the test chosen. We have to note that point-pattern analysis is a descriptive analysis. Even if a particular null model describes our pattern well, it is not appropriate to conclude that the mechanism behind the null model is the mechanism responsible for our pattern. Other mechanisms may lead to exactly the same pattern. However, point-pattern analysis helps to characterize our pattern and to put forward hypotheses on the underlying mechanisms that should be tested in subsequent steps in the field.

REFERENCE

1. **Bruno, J. F., Stachowicz, J. J. & Bertness, M. D.**, 2003. Inclusion of facilitation into ecological theory. *Trends in Ecology & Evolution* 18(3): 119-125.
2. **Callaway, R. M. & Walker, L. R.**, 1997. Competition and facilitation: A synthetic approach to interactions in plant communities. *Ecology* 78(7): 1958-1965.
3. **Chave, J.**, 2004. Neutral theory and community ecology. *Ecology Letters* 7(3): 241-253.
4. **Comas, C. & Mateu, J.**, 2007. Modelling forest dynamics: A perspective from point process methods. *Biometrical Journal* 49(2): 176-196.
5. **Condit, R., Ashton, P. S., Baker, P., Bunyavejchewin, S., Gunatilleke, S., Gunatilleke, N., Hubbell, S. P., Foster, R. B., Itoh, A., LaFrankie, J. V., Lee, H. S., Losos, E., Manokaran, N., Sukumar, R. & Yamakura, T.**, 2000. Spatial patterns in the distribution of tropical tree species. *Science* 288(5470): 1414-1418.
6. **Condit, R., Hubbell, S. P. & Foster, R. B.**, 1994. Density-dependence in 2 understorey tree species in a Neotropical forest. *Ecology* 75(3): 671-680.

7. **Connell, J. H.**, 1971. On the role of natural enemies in preventing competitive exclusion in some marine animals and in rain forest trees. *Dynamics of Populations* Pudoc, Wageningen, P. J. den Boer & G. Gradwell: 298-312.
8. **Connell, J. H.**, 1978: Diversity in tropical rain forests and coral reefs-High diversity of trees and corals is maintained only in a non-equilibrium state. *Science* 199(4335): 1302-1310.
9. **Dale, M. R. T., Dixon, P., Fortin, M. J., Legendre, P., Myers, D. E. & Rosenberg, M. S.**, 2002. Conceptual and mathematical relationships among methods for spatial analysis. *Ecography* 25(5): 558-577.
10. **Dickie, I. A., Schnitzer, S. A., Reich, P. B. & Hobbie, S. E.**, 2007. Is oak establishment in old-fields and savanna openings context dependent? *Journal of Ecology* 95(2): 309-320.
11. **Diggle, P. J.** .2003. *Statistical analysis of spatial point patterns*. London, Arnold (Hodder Headline Group).
12. **Franklin, J., Anselin, L. & Rey, S. J.**, 2010. Spatial Point Pattern Analysis of Plants Perspectives on Spatial Data Analysis, Springer Berlin Heidelberg: 113-123.
13. **Getzin, S., Wiegand, K., Schumacher, J. & Gougeon, F. A.**, 2008. Scale-dependent competition at the stand level assessed from crown areas. *Forest Ecology And Management* 255(7): 2478-2485.
14. **Getzin, S., Worbes, M., Wiegand, T. & Wiegand, K.**, 2011. Size dominance regulates tree spacing more than competition within height classes in tropical Cameroon. *Journal of Tropical Ecology* 27: 93-102.
15. **Goreaud, F. & Pelissier, R.** 2003: Avoiding misinterpretation of biotic interactions with the intertype K-12-function: population independence vs. random labelling hypotheses. *Journal of Vegetation Science* 14(5): 681-692.
16. **Haase, P.**, 1995. Spatial pattern-analysis in ecology based on Ripley's K-function: Introduction and methods of edge correction. *Journal of Vegetation Science* 6(4): 575-582.
17. **Halpern, C. B., Antos, J. A., Rice, J. M., Haugo, R. D. & Lang, N. L.**, 2010. Tree invasion of a montane meadow complex: temporal trends, spatial patterns, and biotic interactions. *Journal of Vegetation Science* 21(4): 717-732.
18. **Harms, K. E., Condit, R., Hubbell, S. P. & Foster, R. B.**, 2001. Habitat associations of trees and shrubs in a 50-ha neotropical forest plot. *Journal Of Ecology* 89(6): 947-959.
19. **Hubbell, S. P.**, 2001. *The unified neutral theory of biodiversity and biogeography*. Princeton, Princeton University Press.
20. **Illian, J. & Burslem, D.**, 2007. Contributions of spatial point process modelling to biodiversity theory. *Journal de la société française de statistique* 148(1): 9-29.
21. **Illian, J., Stoyan, D., Stoyan, H. & Penttinen, A.**, 2008. *Statistical Analysis and Modelling of Spatial Point Patterns*. Sussex, Wiley.
22. **Janzen, D. H.**, 1970. Herbivores and the number of tree species in tropical forests. *American Naturalist* 104(940): 501.

23. **Lan, G., Getzin, S., Wiegand, T., Hu, Y., Xie, G., Zhu, H. & Cao, M.**, 2012. Spatial Distribution and Interspecific Associations of Tree Species in a Tropical Seasonal Rain Forest of China. *Plos One* 7(9).
24. **Lan, G., Zhu, H., Cao, M., Hu, Y., Wang, H., Deng, X., Zhou, S., Cui, J., Huang, J., He, Y., Liu, L., Xu, H. & Song, J.**, 2009. Spatial dispersion patterns of trees in a tropical rainforest in Xishuangbanna, southwest China. *Ecological Research* 24(5): 1117-1124.
25. **Law, R., Illian, J., Burslem, D. F. R. P., Gratzler, G., Gunatilleke, C. V. S. & Gunatilleke, I. A. U. N.**, 2009. Ecological information from spatial patterns of plants: insights from point process theory. *Journal Of Ecology* 97(4): 616-628.
26. **Loosmore, N. B. & Ford, E. D.**, 2006. Statistical inference using the G or K point pattern spatial statistics. *Ecology* 87(8): 1925-1931.
27. **Marriott, F. H. C.**, 1979. Barnard's Monte Carlo Tests: How Many Simulations? *Journal of the Royal Statistical Society. Series C (Applied Statistics)* 28(1): 75-77.
28. **Mateu, J.**, 2000. Second-order characteristics of spatial marked processes with applications. *Nonlinear Analysis: Real World Applications* 1(1): 145-162.
29. **Murrell, D. J.**, 2009. On the emergent spatial structure of size-structured populations: when does self-thinning lead to a reduction in clustering? *Journal Of Ecology* 97(2): 256-266.
30. **Nathan, R. & Muller-Landau, H. C.**, 2000. Spatial patterns of seed dispersal, their determinants and consequences for recruitment. *Trends in Ecology & Evolution* 15(7): 278-285.
31. **Perry, G. L. W., Miller, B. P. & Enright, N. J.**, 2006. A comparison of methods for the statistical analysis of spatial point patterns in plant ecology. *Plant Ecology* 187(1): 59-82.
32. **Peters, H. A.**, 2003. Neighbour-regulated mortality: the influence of positive and negative density dependence on tree populations in species-rich tropical forests. *Ecology Letters* 6(8): 757-765.
33. **Ripley, B. D.**, 1976. The Second-Order Analysis of Stationary Point Processes *Journal of Applied Probability* 13(2): 255-266
34. **Sterner, R. W., Ribic, C. A. & Schatz, G. E.**, 1986. Testing for life historical changes in spatial patterns of four tropical tree species. *Journal Of Ecology* 74(3): 621-633.
35. **Stoyan, D. & Penttinen, A.**, 2000. Recent applications of point process methods in forestry statistics. *Statistical Science* 15(1): 61-78.
36. **Stoyan, D. & Stoyan, H.**, 1994. *Fractals, random shapes, and point fields: Methods of geometrical statistics*. Chichester, John Wiley & Sons.
37. **Tilman, D.**, 1994. Competition and biodiversity in spatially structure habitats. *Ecology* 75(1): 2-16.
38. **Wiegand, K., Jeltsch, F. & Ward, D.**, 2000. Do spatial effects play a role in the spatial distribution of desert-dwelling *Acacia raddiana*? *Journal of Vegetation Science* 11(4): 473-484.

39. **Wiegand, T., Gunatilleke, C. V. S., Gunatilleke, I. A. U. N. & Huth, A.**, 2007. How individual species structure diversity in tropical forests. *Proceedings of the National Academy of Sciences of the United States of America* 104(48): 19029-19033.
40. **Wiegand, T., Gunatilleke, S. & Gunatilleke, N.**, 2007. Species associations in a heterogeneous Sri lankan dipterocarp forest. *American Naturalist* 170(4): E77-E95.
41. **Wiegand, T. & Moloney, K. A.**, 2004. Rings, circles, and null-models for point pattern analysis in ecology. *Oikos* 104(2): 209-229.
42. **Yamada, I. & Rogerson, P. A.**, 2003. An empirical comparison of edge effect correction methods applied to K-function analysis. *Geographical Analysis* 35(2): 97-109.

CÁC PHƯƠNG PHÁP PHÂN TÍCH MÔ HÌNH ĐIỂM KHÔNG GIAN ỨNG DỤNG TRONG SINH THÁI RỪNG

Nguyễn Hồng Hải
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TÓM TẮT

Rất nhiều các phương pháp phân tích về mô hình điểm đã được phát triển trong nhiều lĩnh vực khoa học. Thống kê bậc nhất mô tả sự biến động của mật độ các điểm trên phạm vi lớn của vùng nghiên cứu, trong khi tính chất của bậc hai là tổng hợp thống kê của khoảng cách giữa các điểm và cung cấp khả năng nhận dạng các loại mô hình ở các phạm vi khác nhau. Thống kê bậc hai dựa vào hàm Ripley's K được sử dụng rộng rãi trong sinh thái để mô tả mô hình không gian và để phát triển giả thuyết về quá trình đang diễn ra. Mục tiêu của bài báo này là thống kê lại các phương pháp phân tích mô hình điểm không gian có thể ứng dụng trong sinh thái rừng. Chúng tôi đã tổng hợp các vấn đề liên quan trong phần thảo luận và kết luận. Chúng tôi cũng giới thiệu phần mềm Programita để ứng dụng tất cả các phương pháp được trình bày trong bài báo.